Interlayer Josephson coupling in stripe-ordered superconducting cuprates

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Motivated by experiments on $La_{2-x}Ba_xCuO_4$ which suggest that stripe order coexists with two-dimensional pairing without interlayer phase coherence, we determine the interlayer Josephson coupling in the presence of stripe order. We employ a mean-field description of bond-centered stripes, with a zero-momentum superconducting condensate and alternating stripe directions pinned by the low-temperature tetragonal (LTT) lattice structure. We show that the Fermi-surface reconstruction arising from strong stripe order can suppress the Josephson coupling between adjacent layers by more than an order of magnitude.

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I. INTRODUCTION

Stripe order is a fascinating phenomenon in cuprate superconductors.[1,2] Originally detected in neutron-scattering experiments on $La_{2-x-y}Nd_ySr_xCuO_4$,[3] this combination of unidirectional spin and charge order was found in other members of the "214" cuprate family as well.[2] Incommensurate low-energy spin fluctuations, often interpreted as precursor to stripe order, are seen not only in $La_{2-x}Sr_xCuO_4$, but also in $YBa_2Cu_3O_{6+\delta}$ (Refs. 4 and 5) and $Bi_2Sr_2CaCu_2O_{8+\delta}$.[6] Together with STM measurements, which detected signatures of charge stripes (albeit with substantial disorder) on the surface of $Bi_2Sr_2CaCu_2O_{8+\delta}$ and $Ca_{2-x}Na_xCuO_2Cl_2$,[7] these findings suggest that the tendency toward stripes is common to underdoped cuprates.

What is less clear is the role of stripes for superconductivity. A large body of experiments appears consistent with the concept of competing superconducting and magnetic order parameters,[2] including e.g., the magnetic-field enhancement of spin-density wave (SDW) order. However, a few observations also point to a cooperative interplay of SDW and pairing. In $La_{15/8}Ba_{1/8}CuO_4$, the onset of SDW order upon decreasing temperature is accompanied by a significant drop in the in-plane resistivity, while bulk Meissner effect sets in at much lower temperatures.[8,9] This intermediate-temperature regime of $La_{15/8}Ba_{1/8}CuO_4$ has been interpreted in terms of fluctuating $2d$ pairing, without interlayer phase coherence. (A related phenomenon is the suppression of the Josephson plasma resonance seen in the optical-conductivity measurements of $La_{2-x}Sr_xCuO_4$ upon application of a moderate c-axis field.[10]) In order to explain the absence of an effective interlayer coupling, the existence of a stripe-modulated (i.e., finite-momentum) superconducting condensate, a so-called pair density wave (PDW), was postulated.[11,12] Indeed, in the absence of a uniform condensate, the lowest-order Josephson coupling between neighboring layers vanishes, if the condensate modulation direction alternates from layer to layer—the latter being the result of the in-plane lattice distortions inherent to the LTT structure. However, some properties of the PDW state are not easily compatible with experiments: both photoemission and STM have established a d-wave like gap in the stripe-ordered state above T_c, whereas the PDW state has a full Fermi surface.

In this paper, we shall investigate whether a primarily uniform condensate in a stripe-ordered state could be compatible with the scenario of fluctuating $2d$ pairing in $La_{15/8}Ba_{1/8}CuO_4$, i.e., the absence of interlayer phase coherence. To this end, we calculate the interlayer Josephson coupling for a mean-field model of superconducting charge-density wave (CDW) state with realistic parameter values. The stripe order induces a reconstruction of the Fermi surface, with rotation symmetry breaking in each CuO_2 plane. As the stripe direction alternates from layer to layer, the reconstructed Fermi surfaces of adjacent layers do not match in momentum space. This effect leads to a significant suppression of the Josephson coupling;[13] an additional suppression arises from incommensurate antiferromagnetism of realistic amplitude.

II. ORDER PARAMETERS AND MEAN-FIELD MODEL

We start by enumerating the relevant order parameters for a superconducting stripe state. Charge and spin density waves, with wave vectors $\vec{Q}_c$ and $\vec{Q}_s$, are related to expectation values of particle-hole bilinears in the singlet and triplet channel, respectively. For instance, a CDW is characterized by nonzero $F_c(\vec{k})=\Sigma_\sigma\langle c^\dagger_{\vec{k}+\vec{Q}_c,\sigma}c_{\vec{k}\sigma}\rangle$ where $c^\dagger_{\vec{k}\sigma}$ creates an electron with momentum $\vec{k}$ and spin σ. The superconducting condensate can have both a uniform component and a modulated (PDW) component with wave vector $\vec{Q}_p$, such that $\langle c_{\vec{k}+\vec{Q}_p,\uparrow}c_{\vec{k}\downarrow}\rangle\neq 0$. On symmetry grounds, a collinear SDW will induce a CDW with wave vector $\vec{Q}_c=2\vec{Q}_s$, a PDW will induce a CDW with $\vec{Q}_c=2\vec{Q}_p$, and in the presence of a uniform condensate a CDW will induce a PDW with $\vec{Q}_p=\vec{Q}_c$.

In our modeling, we start from two CuO_2 layers with homogeneous d-wave pairing and then add CDW/SDW modulations. Each layer $i=1,2$ is described by a quasiparticle model of electrons moving on a square lattice, with the Hamiltonian

$$\mathcal{H}^i=\sum_{\vec{k}\sigma}(\varepsilon_{\vec{k}}-\mu)c^\dagger_{\vec{k}i\sigma}c_{\vec{k}i\sigma}+\mathcal{H}^i_{\mathrm{DW}}+\mathcal{H}^i_{\mathrm{P}} \tag{1}$$

and the in-plane dispersion $\varepsilon_{\vec{k}}=-2t(\cos k_x+\cos k_y)-4t'\cos k_x\cos k_y-2t''(\cos 2k_x+\cos 2k_y)$ with $t'=-t/4$ and $t''=t/12$.

The symmetry-breaking orders are implemented non-self-consistently at the mean-field level by $\mathcal{H}_{\mathrm{DW}}$ and $\mathcal{H}_{\mathrm{P}}$. We

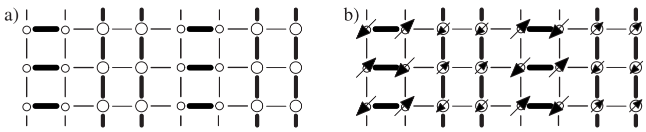

FIG. 1. Schematic real-space structure of valence-bond stripes at doping 1/8 (Ref. 15), with circle sizes (line widths) denoting on-site hole densities (bond strengths). (a) Paramagnetic state with dominant d-wave modulation. (b) Additional spin modulation with antiphase domain walls on hole-rich stripes.

restrict our attention to bond-centered stripes of period 4 (8) in the charge (spin) sector which appear most compatible with experiments at doping 1/8.[1,2] The density-wave part $\mathcal{H}_{\mathrm{DW}}$ is given by

$$\mathcal{H}^i_{\mathrm{DW}} = \sum_{\vec{k}\sigma} \Phi^i_c c^\dagger_{\vec{k}i\sigma} c_{\vec{k}+\vec{Q}^i_{\mathrm{c}},i\sigma} + \Phi^i_s c^\dagger_{\vec{k}i\sigma} c_{\vec{k}+\vec{Q}^i_{\mathrm{s}},i\sigma} + \mathrm{h.c.},$$

$$\Phi^1_c(\vec{k}) = -e^{i\pi/4}\left[\cos\left(k_x + \frac{\pi}{4}\right) - \cos k_y\right]\delta t,$$

$$\Phi^1_s(\vec{k}\sigma) = -\sigma\frac{1}{\sqrt{2}}\frac{1+e^{-i\pi/4}}{1+\sqrt{2}}\delta\mu^\sigma, \tag{2}$$

where $\vec{Q}^1_{\mathrm{c}}=(\pi/2,0)$, $\vec{Q}^1_{\mathrm{s}}=(3\pi/4,\pi)$, and the $\Phi^2_{c,s}$ are obtained from $\Phi^1_{c,s}$ by $x\leftrightarrow y$. Φ_c implements a hopping modulation on the bonds of strength δt, resulting in so-called valence-bond stripes.[14,15] Such bond-charge modulation, Fig. 1(a), is compatible with the STM data of Ref. 7 and implies a strong d-wave component in the form factor F_c of the CDW order parameter. The associated collinear spin order, Fig. 1(b), is implemented via a spin-dependent chemical potential $\delta\mu^\sigma$ in Φ_s.[16]

The pairing part $\mathcal{H}_{\mathrm{P}}$ is dominated by a uniform $d_{x^2-y^2}$-wave pairing mean field Δ_0. In addition, a pairing modulation of amplitude $\delta\Delta$ is assumed along with the CDW, resulting in a pattern of bond pairing amplitudes qualitatively similar to Fig. 1(a), but with d-wave sign structure.

$$\mathcal{H}^i_{\mathrm{P}} = \sum_{\vec{k}} \Delta_{\vec{k}} c^\dagger_{\vec{k}i\uparrow} c^\dagger_{-\vec{k}i\downarrow} + \Phi^i_p c^\dagger_{\vec{k}i\uparrow} c^\dagger_{-\vec{k}-\vec{Q}_{\mathrm{c}},i\downarrow} + \Phi^{i*}_p c^\dagger_{\vec{k}+\vec{Q}^i_{\mathrm{c}},i\uparrow} c^\dagger_{-\vec{k}i\downarrow} + \mathrm{h.c.},$$

$$\Delta_{\vec{k}} = \Delta_0(\cos k_x - \cos k_y),$$

$$\Phi^1_p(\vec{k}) = -e^{i\pi/4}\frac{1}{2}\left[\cos\left(k_x + \frac{\pi}{4}\right) + \cos k_y\right]\delta\Delta, \tag{3}$$

and Φ^2_p is obtained from Φ^1_p by $x\leftrightarrow y$.[17]

III. INTERLAYER JOSEPHSON COUPLING

A. Interlayer tunneling

The Hamiltonian of the full system of two adjacent CuO_2 layers is given by

$$\mathcal{H} = \mathcal{H}^1 + \mathcal{H}^2 + \sum_{\vec{k}\sigma}[t_\perp(\vec{k}) c^\dagger_{\vec{k}1\sigma} c_{\vec{k}2\sigma} + \mathrm{h.c.}] \tag{4}$$

with $\mathcal{H}^i$ given in Eq. (1), and the interlayer hopping matrix element[19]

$$t_\perp(\vec{k}) = \frac{t_\perp}{4}(\cos k_x - \cos k_y)^2. \tag{5}$$

To calculate the Josephson coupling it is convenient to multiply global phase factors θ_i to the superconducting mean fields Δ_0 and $\delta\Delta$ in layer i. Then, the Josephson coupling measures the interplane phase stiffness,

$$J_J = \frac{1}{2}[F(\delta\theta=\pi) - F(\delta\theta=0)] \tag{6}$$

where $F=F^1+F^2+\delta F(\delta\theta)$, F^i is the free energy of the isolated layer i, δF is the interlayer tunneling contribution to the free energy, and $\delta\theta=\theta_2-\theta_1$.

Assuming $t_\perp \ll t$, δF can be determined in second-order perturbation theory in $t_\perp$,

$$\delta F = \frac{1}{\beta}\mathrm{tr}(\hat{\mathcal{G}}^1\hat{T}\hat{\mathcal{G}}^2\hat{T}) = \frac{1}{N}\sum_{\vec{k}}\delta F_{\vec{k}},$$

$$\delta F_{\vec{k}} = \frac{1}{\beta}\sum_{\omega_n} t_\perp(\vec{k})^2 \sum_{\alpha,\beta=0}^{1}(-)^{\alpha+\beta}\mathcal{G}^{1,\alpha\beta}_{\vec{k}n}\mathcal{G}^{2,\beta\alpha}_{\vec{k}n}, \tag{7}$$

where $\mathcal{G}^i$ is the full Green's operator on layer i, $\hat{T}$ is the interlayer tunneling operator from $t_\perp$, β is the inverse temperature, and N the number of unit cells. The indices α,β denote particle-hole space. Since the stripe directions are orthogonal, we have to consider the full (nonreduced) Brillouin zone (BZ). For a period-4 CDW (period-8 CDW+SDW) in each superconducting layer, the calculation of δF involves

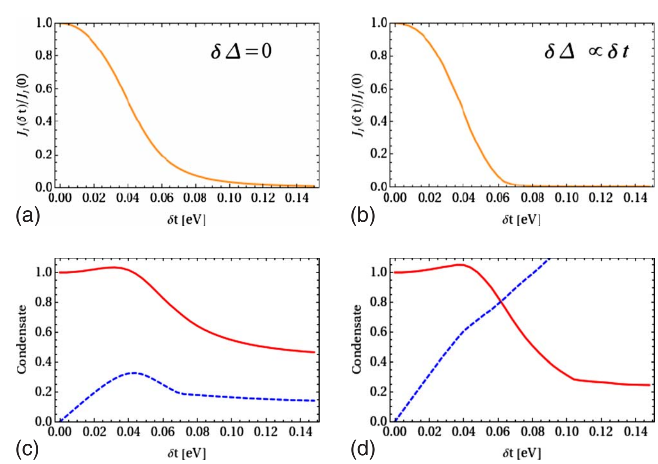

FIG. 2. (Color online) Interlayer Josephson coupling J_J (a,b) and superconducting condensate amplitudes ψ_0 (solid) and $\psi_{\vec{Q}_c}$ (dashed) (c,d) as functions of the hopping modulation strength δt, at fixed doping $x=1/8$ and in the absence of magnetic order. The modulation in the pairing field is $\delta\Delta/\delta t=0$ (a,c) and 1.3 (b,d). The couplings and condensates are normalized wrt the values of J_J and ψ_0 at $\delta t=\delta\Delta=0$.

the diagonalization of a 8×8 (16×16) Hamiltonian matrix to construct the Green's functions required in Eq. (7).

B. Results

The numerical calculations have been performed at zero temperature, with parameters $t=0.15$ eV, $\Delta_0=0.024$ eV, and fixed doping $x=1/8$. For the homogeneous case, this corresponds to $\mu=-0.126$ eV.

Results for J_J for charge-only stripes as function of the hopping modulation δt ($\delta\Delta=\delta\mu^\sigma=0$) are shown in Fig. 2(a). For large modulation amplitude, the Josephson coupling is seen to be strongly suppressed, e.g., by roughly a factor 10 for $\delta t=0.07$ eV. A simultaneous modulation in the condensate mean field by $\delta\Delta$ (here chosen to produce similar relative modulation strengths in the resulting bond kinetic energies and pairings) suppresses the Josephson coupling even further, Fig. 2(b).

Figures 2(c) and 2(d) show the corresponding evolution of the homogeneous and modulated condensate amplitudes, ψ_0 and $\psi_{\vec{Q}_c}$, calculated from the solution of the mean-field Hamiltonian (1). Here, we define ψ as the sum of the magnitudes of the s-wave (on-site), $s_{x^2+y^2}$-wave and $d_{x^2-y^2}$-wave condensates calculated from the real-space pairing amplitudes extracted from the solution of $\mathcal{H}^i$ (note that the s and $d_{x^2-y^2}$ representations of the point group mix in the presence of stripe order). A modulated condensate $\psi_{\vec{Q}_c}$ is always present for $\delta t\neq0$, but remains small if $\delta\Delta=0$. In contrast, for $\delta\Delta\propto\delta t$ as in Fig. 2(d), $\psi_{\vec{Q}_c}$ increases and eventually dominates over ψ_0. A comparison between the evolution of J_J and ψ_0 reveals that in the range of δt where J_J drops dramatically, the uniform condensate ψ_0 displays a much weaker depletion. This is also true for panels (b) and (d) where, at $\delta t\approx0.08$ eV, J_J is reduced by a factor 200 while ψ_0 still has half of its original value. From this we conclude that the primary source of the suppression of the Josephson coupling in our calculation is different from that of the PDW proposal by Berg *et al.*[12] where the layer decoupling is due to the *absence* of a homogeneous condensate.

Analyzing our results further, we identify the momentum-space mismatch of the Fermi surfaces of the two layers, arising from the orthogonal stripe modulation, as the main source of the suppression of J_J. This is also accompanied by a small mismatch of the nodal lines of the superconducting order parameter in the two layers, due to broken rotational symmetry in each layer.[17] These effects can be nicely seen in the momentum-resolved contributions $J_J(\vec{k})=\delta F_{\vec{k}}(\delta\theta=\pi)-\delta F_{\vec{k}}(\delta\theta=0)$ to the Josephson coupling, shown in Fig. 3. In the homogeneous case, the largest contributions to J_J arise near the antinodal points of the (bare) Fermi surface. These contributions are drastically reduced (note the logarithmic intensity scale) with increasing stripe modulation, as a result of the Fermi-surface distortions accompanying the stripe order, see e.g., Figures 3 and 4 of Ref. 16 and Figs. 2 and 4 of Ref. 18. For sizeable $\delta\Delta$, the combination of Fermi-surface and order-parameter reconstruction even generates regions in momentum space with $J_J(\vec{k})<0$, Fig. 3(b). (For $\delta\Delta=0$, this effect occurs only near the BZ diagonals due to the shift of nodal lines from stripe order, but this has little influence on

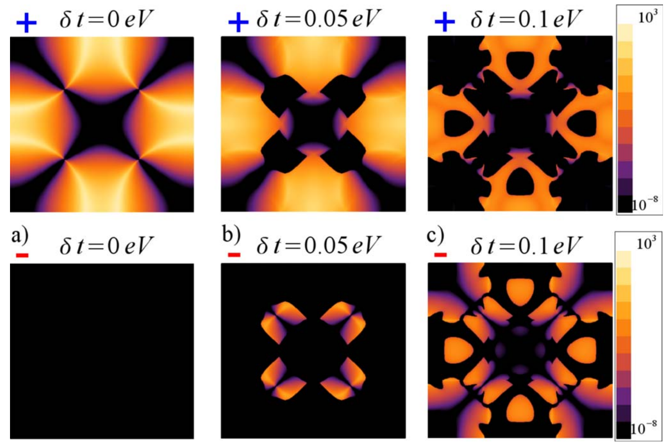

FIG. 3. (Color online) Positive (top) and negative (bottom) momentum-resolved contributions to J_J. Each panel shows $|J_J(\vec{k})/t_\perp^2|$ (see text) as function of $\vec{k}$ on a logarithmic intensity scale. The modulation strength δt is zero in (a) and increases from (b) to (c). The other parameters are as in Fig. 2(b) and 2(d). From (a) to (c) the positive contributions near the antinodal points are reduced, moreover negative contributions appear. Note that the momentum dependence of the interlayer tunneling (5) suppresses the contributions near the diagonals.

J_J due to the specific momentum dependence of the interlayer tunneling.) The imposed pairing modulation $\delta\Delta$ is seen to contribute to the reduction of J_J, Fig. 4(a).

We now turn to the influence of magnetic SDW order as in Fig. 1(b). As shown in Fig. 4(b), SDW order alone (with CDW being parasitic only) leads to a moderate suppression of J_J. Similarly, SDW order in combination with a CDW suppresses J_J further compared to the nonmagnetic case, mainly because of the additional Fermi-surface reconstruction arising from the SDW wave vector. Note that the *relative* spin orientation between the two layers does not enter the result.

Finally, we link our findings to experiments. Unfortunately, the magnitude of the charge modulation in cuprate stripes is not well known: From resonant soft x-ray scattering[20] on $La_{15/8}Ba_{1/8}CuO_4$ the modulation on the oxygens was concluded to be of order factor 4, but the quantitative analysis is model dependent. From the STM data[7] on $Bi_2Sr_2CaCu_2O_{8+\delta}$ and $Ca_{2-x}Na_xCuO_2Cl_2$ one may infer a

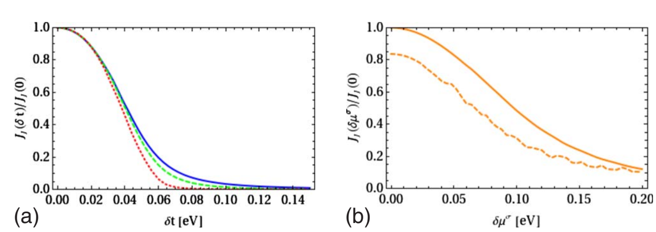

FIG. 4. (Color online) Normalized interlayer Josephson coupling J_J at doping 1/8 for (a) paramagnetic stripes with hopping modulation δt and accompanying pairing modulations $\delta\Delta/\delta t=0$ (solid), 0.5 (dashed), 1.3 (dotted), and (b) antiferromagnetic stripes with magnetic modulation set by $\delta\mu^\sigma$, for $\delta t=\delta\Delta=0$ (solid) and for moderate fixed kinetic-energy modulation of $\approx\pm25\%$ (dashed) achieved via adjusting δt and keeping $\delta\Delta=1.3\delta t$.

typical modulation amplitude in the charge sector of $\pm 20 \ldots 30\%$. In our calculation, a modulation of $\pm 25\%$ in the bond kinetic energies is obtained from $\delta t = 0.023$ eV, which gives a 20% reduction of J_J [Fig. 2(b)], while $\delta t = 0.07$ eV with a factor 90 reduction of J_J corresponds to a kinetic-energy modulation of about a factor 10 (here, the neglect of Mott physics makes this number less meaningful, as the quasiparticle theory does not account for half filling being special).

The magnitude of the magnetic order in striped 214 cuprates is known reasonably well, at $x = 1/8$ the maximum moment size is $50 \ldots 60\%$ of that of the undoped compound.[2,21] In our mean-field calculation, $\delta\mu^\sigma \approx \pm 0.07$ eV corresponds to a maximum moment of $0.36\mu_B$ and gives an additional factor 2 suppression of J_J.

As stripes are particularly stable near doping 1/8, it is conceivable that $La_{15/8}Ba_{1/8}CuO_4$ has rather strong modulation in the charge sector, in which case J_J would be suppressed drastically by stripe order. This in turn would be consistent with the fact that the 2d fluctuating pairing regime is restricted to the vicinity of $x = 1/8$. We refrain from an estimate of how much reduction of J_J is required to match experiments, because corrections beyond the mean-field picture arising from phase fluctuations, quenched disorder, short-range magnetism, and other pseudogap physics are of importance to fully understand the hierarchy of energy scales in $La_{15/8}Ba_{1/8}CuO_4$.

IV. CONCLUSIONS

Using a mean-field quasiparticle framework, we have calculated the interlayer Josephson coupling in superconducting stripe states. We have assumed that the in-plane stripe orientations alternate from layer to layer, as is the case in 214 cuprates with LTT lattice structure. For realistic stripe modulation strengths, we find that the interlayer coupling can be easily reduced by an order of magnitude. The primary cause of this reduction is the momentum-space mismatch between the reconstructed Fermi surfaces of adjacent layers, while the depletion of the zero-momentum superconducting condensate (in favor of a modulated one) is secondary. Whether this effect is sufficient to explain the unusual properties of $La_{15/8}Ba_{1/8}CuO_4$ is not yet clear.

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